June 8, 2001

OU-HET 389

hep-ph/0106086

The matter fluctuation effect to T violation at a neutrino factory

Takahiro Miura*, Tetsuo Shindou†, Eiichi Takasugi‡, and Masaki Yoshimura§

Department of Physics, Osaka University

Toyonaka, Osaka 560-0043, Japan

Abstract

We derived an analytic formula for T violation by using the perturbation theory for small quantities, $\Delta m^2_{21} L/2E$ and $\delta a(x) L/2E$, where $\delta a(x)$ represents symmetric and asymmetric matter fluctuations, i.e., deviations from the average density. We analyzed the effect of matter fluctuations to T violation, by assuming PREM profile of earth matter density. We found that matter fluctuations do not give any viable contribution for $L <$6000km, while the fluctuation effect becomes large due to resonances for $L >$7000km. For 7000km$< L <$8000km, matter fluctuations contribute destructively to the average density term and the net result is small, while for $L >$8000km, the contribution from matter fluctuations becomes large but contributes constructively.

*e-mail address: miura@het.phys.sci.osaka-u.ac.jp

†e-mail address: shindou@het.phys.sci.osaka-u.ac.jp

‡e-mail address: takasugi@het.phys.sci.osaka-u.ac.jp

§e-mail address: masaki@het.phys.sci.osaka-u.ac.jp

1 Introduction

In our previous paper[1], we analyzed the matter fluctuation effect to T violation, $P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$ $(\alpha \neq \beta)$ at a neutrino factory[2] by using the perturbation method developed by Koike and Sato[3] and Ota and Sato[4]. The perturbation is made with respect to small quantities, $\Delta m^2_{21}L/2E$ and $\delta a(x)L/2E$, where $\delta a(x)$ represents matter fluctuations, i.e., deviations from the average density . We examined T violation up to the 2nd order and found that it arises from the average density, the 1st order term which is proportional to $\Delta m^2_{21}L/2E$ and term from matter fluctuations, the 2nd order term proportional to $(\Delta m^2_{21}L/2E)(\delta a(x)L/2E)$. The zeroth order term and terms proportional to $(\Delta m^2_{21}L/2E)^2$ and $(\delta a(x)L/2E)^n$ $(n = 1, 2, 3)$ do not contribute to T violation. The 1st order term and the 2nd order term from symmetric matter fluctuations which we denote δa_s contribute to $\sin\delta$ term and the 2nd order term from asymmetric matter fluctuations, δa_a does to the fake $\cos\delta$ term, where δ is the CP violation phase in MNS neutrino mixing matrix[5].

By using the preliminary reference earth model (PREM)[6] for symmetric matter fluctuations and assuming that asymmetric matter fluctuations are much less than symmetric matter fluctuations given by PREM, we computed T violation and found that the 2nd order term from symmetric and asymmetric matter fluctuations gives only negligible contributions to T violation, and thus the constant (average) matter approximation is valid for $L = 3000$km. On the other hand, for $L = 7332$km, we found that the contribution from symmetric matter fluctuations becomes as large as the 1st order term and moreover they contribute destructively so that T violation becomes very small. This means that the constant (average) matter approximation is not valid for $L >$7000km and also the validity of our 2nd order formula should be examined.

In this paper, we discuss the following three questions: (1) Is the 2nd order formula valid? (2) What is the length where the constant (average) matter approximation fails for T violation? That is, at what length, the matter fluctuation effect becomes important. (3) What is the size of T violation for $L >$7000km?

To answer these questions, we computed the next order (3rd order) contribution to T violation, i.e., the term proportional to $(\Delta m_{21}^2 L/2E)(\delta a(x)_s L/2E)^2$. Our result is as follows: The contribution from matter fluctuation can be safely neglected for L <6000km. When we discuss T violation with length larger than 6000km, the matter fluctuation effect should be taken into account. The 3rd order contribution is negligible in comparison with the 1st and the 2nd order term for all distances. Therefore, T violation can be reliably estimated for all distances and it becomes very small for 7000km< L <8000km. For L >8000km, the 1st and the 2nd terms contribute constructively and T violation becomes large.

This paper is organized as follows: In Sec.2, the analytic formula for T violation is given. The numerical analysis for T violation by using PREM profile is given in Sec.3 and the mechanism how the cancellation occurs for 7000km< L <8000km is explained analytically. The summary is given in Sec.4. The derivation of the analytic formula is given in Appendix.

2 T violation formula

For completeness, we give the 1st and the 2nd order contributions to T violation and the definition of parameters in the formula. We also give the 3rd order contribution from symmetric fluctuations.

(a) Notation

We begin with defining the neutrino mixing matrix as

$$\begin{aligned} U &= e^{i\theta_y\lambda_7}\mathrm{diag}(1,1,e^{i\delta})e^{i\theta_z\lambda_5}e^{i\theta_x\lambda_2} \\ &= \begin{pmatrix} c_x c_z & s_x c_z & s_z \\ -s_x c_y - c_x s_y s_z e^{i\delta} & c_x c_y - s_x s_y s_z e^{i\delta} & s_y c_z e^{i\delta} \\ s_x s_y - c_x c_y s_z e^{i\delta} & -c_x s_y - s_x c_y s_z e^{i\delta} & c_y c_z e^{i\delta} \end{pmatrix}, \end{aligned} \tag{1}$$

where λ_j ($j = 2, 5, 7$) are Gell-Mann matrices and $c_a = \cos\theta_a$ and $s_a = \sin\theta_a$. Since the Majorana CP-violation phases are irrelevant to the neutrino oscillations (flavor oscillations)[7], we neglected them. The relation between the flavor eigenstates, $|\nu_\alpha\rangle$ ($\alpha =$

e, μ, τ), and the mass eigenstates, $|\nu_i\rangle$ ($i = 1, 2, 3$), is given by

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle \ . \tag{2}$$

The evolution of the flavor eigenstates in matter with energy E is given by

$$i\frac{d}{dx}|\nu_\beta(x)\rangle = H(x)_{\beta\alpha}|\nu_\alpha(x)\rangle \ , \tag{3}$$

where Hamiltonian $H(x)_{\beta\alpha}$ is given by

$$H(x)_{\beta\alpha} = \frac{1}{2E}\left\{U_{\beta i}\begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix}_{ii} U_{i\alpha}^\dagger + \begin{pmatrix} a(x) & & \\ & 0 & \\ & & 0 \end{pmatrix}_{\beta\alpha}\right\} . \tag{4}$$

Here $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ with m_i being the mass of $|\nu_i\rangle$, G_F is the Fermi coupling constant and

$$a(x) \equiv 2\sqrt{2}G_F n_e(x)E = 7.56 \times 10^{-5}\left(\frac{\rho(x)}{\mathrm{g/cm^3}}\right)\left(\frac{Y_e}{0.5}\right)\left(\frac{E}{\mathrm{GeV}}\right)\mathrm{eV}^2 \ , \tag{5}$$

where $n_e(x)$, Y_e and $\rho(x)$ are the electron number density, the electron fraction and the matter density, respectively. For the electron fraction, we use $Y_e = 0.5$.

We separate the matter density fluctuation from its average $\bar{a}$,

$$\delta a(x) \equiv a(x) - \bar{a} \ , \tag{6}$$

and consider the deviation $\delta a(x)$ as a perturbative term. That is, we solve the evolution equation by treating $\delta a(x)L/2E$ and $\Delta m_{21}^2 L/2E$ as perturbative terms, because they are small for most of the cases of planned neutrino factories.

T violation is defined by

$$\Delta P^T_{\nu_\alpha\nu_\beta} = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha) \ , \tag{7}$$

which is evaluated by using the method developed by Koike and Sato[3], Ota and Sato[4]. Ota and Sato showed that the 1st order approximation for $\delta a(x)L/2E$ is good enough to reproduce the transition probability. However, T violation is as small as a few % of the transition probability so that this approximation is not valid. In fact, we showed the 2nd order term becomes as important as the 1st order term for L >6000km. We show T violation, $\Delta P^T_{\nu_e\nu_\mu}$ up to the 3rd order perturbation for symmetric matter fluctuations and the 2nd order for asymmetric matter fluctuations. The analytical formula is given by expanding symmetric and asymmetric matter fluctuations in terms of Fourier series as

$$\delta a(x)_s = \sum_{n=\pm 1,\pm 2,\cdots} a_{2n} e^{-iq_{2n}x} \ , \quad \delta a(x)_a = \sum_{n=0,\pm 1,\cdots} a_{2n+1} e^{-iq_{2n+1}x} \ , \tag{8}$$

where

$$q_n = \frac{n\pi}{L} \ . \tag{9}$$

(b) The analytic formula for T violation

In order to define T violation, we define the following quantities:

$$\begin{aligned}
\tan 2\theta_{\tilde{z}} &= \frac{s_{2z}(\Delta m^2_{31} - \Delta m^2_{21} s^2_x)}{c_{2z}(\Delta m^2_{31} - \Delta m^2_{21} s^2_x) - \bar{a}} \ , \\
\lambda_\pm &= \frac{1}{2}\Big(\Delta m^2_{31} + \Delta m^2_{21} s^2_x + \bar{a} \\
&\quad \pm \sqrt{\{c_{2z}(\Delta m^2_{31} - \Delta m^2_{21} s^2_x) - \bar{a}\}^2 + s^2_{2z}(\Delta m^2_{31} - \Delta m^2_{21} s^2_x)^2}\Big) \ , \\
k_1 &= \frac{\Delta m^2_{21} c^2_x - \lambda_-}{2E} \ , \\
k_2 &= \frac{\lambda_+ - \Delta m^2_{21} c^2_x}{2E} \ , \\
k &= \frac{\lambda_+ - \lambda_-}{2E} \ .
\end{aligned} \tag{10}$$

The sum of the 1st and the 2nd order terms due to symmetric matter fluctuations contributes to $\sin\delta$ term and is given by

$$\begin{aligned}
\Delta P^{T(1+2s)}_{\nu_e\nu_\mu} &= -\frac{\Delta m^2_{21}}{E} s_{2x} s_{2y} s_{2\tilde{z}} s_\delta \left[\frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} + \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2}\right] \sin\frac{k_1 L}{2} \sin\frac{k_2 L}{2} \sin\frac{kL}{2} \\
&\times \left[1 + 2\sum_{n=1,2,\cdots} \frac{a_{2n}}{2E}\left(\frac{c^2_{\tilde{z}} k_1}{k_1^2 - q_{2n}^2} - \frac{s^2_{\tilde{z}} k_2}{k_2^2 - q_{2n}^2} + \frac{c_{2\tilde{z}} k}{k^2 - q_{2n}^2}\right)\right] \ .
\end{aligned} \tag{11}$$

The asymmetric matter fluctuation contributes to the $\cos\delta$ term and is given by

$$\begin{aligned} \Delta P^{T(2a)}_{\nu_e\nu_\mu} &= -\frac{\Delta m^2_{21}}{E} s_{2x}s_{2y}s_{2\tilde{z}}c_\delta \left[\frac{c_{\tilde{z}}c_{z-\tilde{z}}}{k_1}+\frac{s_{\tilde{z}}s_{z-\tilde{z}}}{k_2}\right] \sin\frac{k_1L}{2}\sin\frac{k_2L}{2}\sin\frac{kL}{2} \\ &\times \left\{ \sum_{n=0,1,\cdots} \frac{a_{2n+1}}{2E}\frac{k}{k^2-q^2_{2n+1}}\cot\frac{kL}{2} - \frac{c^2_{\tilde{z}}k_1}{k_1^2-q^2_{2n+1}}\cot\frac{k_1L}{2} \right. \\ &\left. -\frac{s^2_{\tilde{z}}k_2}{k_2^2-q^2_{2n+1}}\cot\frac{k_2L}{2} \right\} . \end{aligned} \tag{12}$$

The 3rd order contribution from symmetric fluctuations is given by

$$\begin{aligned} \Delta P^{T(3s)}_{\nu_e\nu_\mu} &= -\frac{\Delta m^2_{21}}{E} s_{2x}s_{2y}s_{2\tilde{z}}s_\delta \left[\frac{c_{\tilde{z}}c_{z-\tilde{z}}}{k_1}+\frac{s_{\tilde{z}}s_{z-\tilde{z}}}{k_2}\right] \\ & \left(\left\{ \sum_{n,m=\pm1,\pm2,\cdots} \frac{a_{2n}a_{2m}}{(2E)^2} \left[\frac{c_{2\tilde{z}}}{k+q_{2n}} \left(\frac{c^2_{\tilde{z}}}{k_1+q_{2m}} - \frac{s^2_{\tilde{z}}}{k_2-q_{2m}} \right) \right.\right.\right. \\ & + \left(\frac{c^2_{\tilde{z}}}{k_1+q_{2n}} - \frac{s^2_{\tilde{z}}}{k_2-q_{2n}} \right) \left(\frac{c^2_{\tilde{z}}}{k_1+q_{2n+2m}} - \frac{s^2_{\tilde{z}}}{k_2-q_{2n+2m}} \right) \\ & + \frac{1}{k+q_{2n}} \left(\frac{c^2_{\tilde{z}}}{k_1+q_{2n+2m}} - \frac{1}{2}\frac{s^2_{\tilde{z}}}{k_2-q_{2n}} \right) \Bigg) \Bigg\} \sin\frac{k_1L}{2}\sin\frac{k_2L}{2}\sin\frac{kL}{2} \\ & + \frac{s^2_{2\tilde{z}}}{4} \left(\sum_{n=\pm1,\pm2,\cdots} \frac{a_{2n}a_{-2n}}{(2E)^2}\frac{L}{k-q_{2n}} \right) \left(\sin\frac{k_1L}{2}\sin\frac{k_2L}{2}\cos\frac{kL}{2} + \frac{1}{2}\sin^2\frac{kL}{2} \right) \Bigg) . \end{aligned} \tag{13}$$

It is amusing to see that the 1st and the 2nd order terms due to symmetric matter fluctuations have the same coefficient and the same oscillation term as we can see from Eq.(11). If one of resonance conditions, $k_1^2 = q_2^2$, $k_2^2 = q_2^2$ and $k^2 = q_2^2$ is realized at some distance, the matter fluctuation term dominates over the 1st order term, although this singular behavior is cancelled by the oscillation term. The asymmetric matter contribution gives the similar contribution except for the angle δ. The 3rd order term is rather complicated and seems to have double poles when $n = m$, which is false due to the cancellation between other terms, and there are no singularities.

As we expected, symmetric and asymmetric matter fluctuations contribute to the $\sin\delta$ and $\cos\delta$ parts of T violation.

The important fact for T violation is that the relation

$$\Delta P^T_{\nu_e\nu_\mu} = \Delta P^T_{\nu_\mu\nu_\tau} = \Delta P^T_{\nu_\tau\nu_e} , \tag{14}$$

holds even in our 3rd order formula. That is, we can not gain any further information by examining other channels. This fact for the constant matter was first found by Krastev and Petcov[8] and the validity for the 2nd order formula is proved in Ref.1.

3 Numerical analysis

By using the analytic formula, $\Delta P^{T(1+2s+2a+3s)}_{\nu_e\nu_\mu}$, we investigate the L and E dependences of T violation. The energy of neutrino, E and the distance, L are taken from 1GeV to 30GeV and from 1000km to 12000km. Neutrino oscillation parameters are taken as

$$\begin{aligned} \Delta m^2_{31} &= 3\times 10^{-3}\ (\mathrm{eV}^2)\ , \quad \Delta m^2_{21} = 5\times 10^{-5}\ (\mathrm{eV}^2)\ , \\ \sin 2\theta_x &= \sin 2\theta_y = 1\ , \quad \sin\theta_z = 0.1\ , \quad \delta = \pi/4\ , \end{aligned} \tag{15}$$

as a typical case.

(a) General features of T violation

Here we examine the effect of symmetric fluctuations to T violation. For the average density and symmetric fluctuations, we assume PREM and decompose it into the average density $\bar{a}$ and the Fourier coefficients a_{2n} $(n = 1, 2, 3, 4)$, which are functions of L. It is a general belief that the average density is determined with better accuracy than matter fluctuations.

We computed T violation, $\Delta P^T_{\nu_e\nu_\mu}$ from L =1000km to 12000km. We found that the 2nd and the 3rd order terms are not important for $L <$ 6000km and thus the constant (average) matter approximation is valid. We remind that the 1st order term contains the effect of the constant matter, while the 2nd and the 3rd order terms are due to matter fluctuations as we can see in Eqs.(11), (12) and (13).

Now we concentrate on the distance $L >$ 6000km. In Figs.1-6, we show the E dependence of T violation for L =6000km, 7000km, 7700km, 8000km, 10000km and 11000km, where the 1st, the 2nd and the 3rd order terms are shown by the dashed line, the dotted line and the dash-dotted line, respectively. The solid line represents the sum of them. The vacuum (no matter) case is shown by the dash-twodotted line for comparison.

When L >7000km, the 2nd order term becomes the same order as the 1st order term and is no more neglected. Moreover, as we see in Figs.2 and 3, for 7000km< L <8000km, the 1st and the 2nd order terms contribute destructively and T violation becomes small. When 8000km< L <9000km, the 1st and the 2nd order terms contribute constructively as we see in Fig.4. For L >10000km, the 2nd order term becomes much larger than the 1st order term.

For all distances, the 3rd order term (the dash-dotted line) gives only a negligible contribution and it is hard to see from these figures. Thus, the 3rd order term can safely be neglected so that we need to consider the 1st and the 2nd order terms only.

When we see Figs.1-6, for $E > 5$GeV, we observe that there are two peaks. One is at E =5.5GeV and the other is at E =10GeV for L =6000km. The energies of these peaks increase as L increases. T violation for energies larger than that of the lower energy peak lies between these two peak values.

We examined the L dependence of these two peak values and the result is shown in Fig.7a. Faint diamonds and dark diamonds correspond to the peak values of the lower and the higher energy peaks, respectively. Diamonds, stars, boxes, triangles, circle show T violation for $s_z = \sin\theta_{13} =$ 0.1, 0.08, 0.06, 0.04 and 0.02. Fig.7b shows the case for E =30GeV.

Due to the cancellation between the 1st and the 2nd order terms, T violation becomes zero at around 7300km. For E =30GeV, there are two zeros at around L =7300km and 9600km.

Another aspect we can observe from these figures is that the s_z dependences of T violation is roughly linear.

(b) The relative sign between the 1st and the 2nd order terms

In order to see the changes of the relative sign between the 1st and the 2nd order terms, we write these terms omitting the overall factor and the oscillation term as

$$1 + 2\sum_{n=1,2,\cdots} \frac{a_{2n}}{2E}\left(\frac{c_{\tilde{z}}^2 k_1}{k_1^2 - q_{2n}^2} - \frac{s_{\tilde{z}}^2 k_2}{k_2^2 - q_{2n}^2} + \frac{c_{2\tilde{z}} k}{k^2 - q_{2n}^2}\right) . \tag{16}$$

The 1st order term is expressed by 1, while the 2nd order term is expressed by the sum of Fourier coefficients, a_{2n}. The singularities at $k_1^2 = q_{2n}^2$, $k_2^2 = q_{2n}^2$ and $k^2 = q_{2n}^2$ correspond to resonances. In general, the 2nd order term is smaller than the 1st order term if k_1, k_2 and k are away from resonance points. Since $q_{2n} = 2n\pi/L$, we compared $(k_1L/2)^2$, $(k_2L/2)^2$ and $(kL/2)^2$ with $(n\pi)^2$ for E >5GeV in Fig.8. For L is as small as 3000km, the resonance condition is not satisfied so that the 2nd term gives only a negligible contribution.

We consider the change of the relative sign between the 1st and the 2nd order terms as L increases from 7000km to 8000km at the lower energy peak position. Since $a_2 < 0$, $k_1 < 0$, $k_2 > 0$ and $k > 0$, the sign of the k_1 term is negative for L=7000km and becomes positive at L =8000km. The resonance occurs at around L =7700km. The same is true for k_2 term. The k term never reaches to the resonance point. That is, when L is as small as 6000km, all terms can be neglected because they never approach to the resonance point. As a result, T violation is positive. When L exceeds 7000km, contributions from the k_1 and k_2 terms become important and their signs are negative. As L approaches to 7700km, their contribution cancelles the 1st order term and T violation vanishes at around 7550km. Between 7550km< L <7700km, both k_1 and k_2 terms become still negative and dominates over the 1st term so that T violation becomes negative. At the resonance point, this singularity is cancelled by the oscillation term and T violation obtains non-zero negative value. After passing the resonance point, both k_1 and k_2 terms become positive and contribute additively to the 1st order term. Since the oscillation term becomes negative and thus T violation remains to be negative.

The L dependence of T violation at the larger energy can be understood similarly. For E =30GeV, the zero of T violation at around 7300km can be understood similarly. The zero for 9600km is due to the resonance for k.

(c)The effect from the uncertainty for the average matter density

So far, we used PREM to derive the average density and the matter fluctuations. For the sake of argument, we consider 5% uncertainty for the average matter density although we do expect that the uncertainty is much less. We examined how T violation changes when the average density is changed by 5%. If the average matter density changes, it affects to the angle $s_{\tilde{z}}$ and k_1, k_2 and k as we can see in Eqs.(10). As a result, the distance L where the resonance occurs changes. In Fig.9, we show the L dependence of T violation. Diamonds, boxes and stars correspond to the average value from PREM, 5% smaller value and 5% larger value. Faint and dark ones correspond the lower energy peak and the higher energy peak. It is interesting to see that the zero point shift to the longer (shorter) L by about 200km as the average density becomes smaller (larger).

(d) The effect from the uncertainty of matter fluctuations

As we discussed in the subsection (b), the main contribution from matter fluctuations is from the term containing the Fourier coefficient a_2 for symmetric fluctuations, and similarly a_1 for asymmetric fluctuations. Since a_2 is determined from the most dense matter part (middle part) along the neutrino path. On the other hand, higher modes a_{2n} $(n = 2, 3, \cdots)$ are determined mainly by the crust of earth. Therefore, for $L > 6000$km, a_2 is considered to be rather unambiguously determined reflecting the deep inside structure of mantle.

For asymmetric fluctuations, a_1 reflects the global asymmetric feature of matter profile. For distances at a neutrino factory, neutrinos pass mainly through the mantle and we do not expect much uncertainty a_1. For the shorter length, neutrinos pass through the crust and sometimes the sea. In this situation, the matter profile with large asymmetric matter fluctuations may need to be considered.[9]

For the sake of argument, we assume that there is about 10% uncertainty for a_2, i.e., $a_{2n} = (1 \pm 0.1)(a_{2n})_{PREM}$. Since $\Delta P^{T(2s)}_{\nu_\alpha \nu_\beta}$ depends linearly on a_2, the 10% uncertainty for a_2 gives the same uncertainty for $\Delta P^{T(2s)}_{\nu_\alpha \nu_\beta}$. For distances where the 2nd order term is neglected, the uncertainty from symmetric fluctuations is negligible ($L <$6000km). In distances where the 2nd order term dominates, then 10% uncertainty appears ($L >$ 10000km). The uncertainty for $L >$ 8000km is smaller than 10%. For 7000km$<$ $L <$8000km, the uncertainty is larger than 10%. In Fig.10, we show how T violation becomes uncertain if a_2 has 10% uncertainty.

For asymmetric fluctuations, we assumed that $a_{2n-1} = 0.1(a_{2n})_{PREM}$ in addition to symmetric matter fluctuations determined from PREM. The effect from asymmetric fluctuations is very similar to the case of symmetric fluctuations and it is shown in Fig.10.

4 Summary

In this paper, we derived the analytic formula for T violation up to the 3rd order term for small quantities, $\Delta m_{21}^2 L/2E$ and $\delta a(x)L/2E$. By using this formula, we examined the E and L dependence of T violation.

We showed that the effect from both symmetric and asymmetric matter fluctuations are negligible for L <6000km and the constant (average) matter approximation (the 1st order term) is valid. Therefore, T violation contains the uncertainty from the average matter aside from mixing angles. Since the average is considered to be determined with much less uncertainty than matter fluctuations, T violation is determined with a good accuracy for L <6000km.

For $L >$ 6000km, the situation changes. Matter fluctuations (the 2nd order term) give a sizable effect to T violation. Moreover, the 1st and the 2nd order terms contribute destructively for 7000km< L <7700km. As a result, the T violation becomes very small.

In Fig.7, we showed the L dependence of T violation at the lower energy peak (the peak at E =5.5GeV for L =6000km) and the higher energy peak (the peak for E =10GeV for L =6000km). T violation at the lower energy peak has the largest value for 5000km< L <6000km and L ∼10000km and T violation at the higher energy peak does for 3000km< L <4000km. T violation becomes zero at around L =7300km. This is due to the resonance effect, which we explained why this happens by using the analytic formula. For E = 30GeV, T violation behaves similarly and has the largest value at 5000km.

We also examined how the L dependence of T violation varies as the average matter is changed by ±5%, although we believe that the average matter density is determined with much less uncertainty. We found that the distance which gives zero of T violation shifts about $\mp$200km.

Acknowledgment This work is supported in part by the Japanese Grant-in-Aid for Scientific Research of Ministry of Education, Culture, Sports, Science and Technology, No.12047218.

Appendix A : The brief summary of our previous result

In Ref.1, we derived T violation fromula up to the 2nd order with respect to small quantities of $\Delta m^2_{21}L/4E$ and $\delta aL/4E$, where δa represents matter fluctuations, where both symmetric δa_s and asymmetric δa_a fluctuations are taken into account. Here, we give the brief summary of the previous result and the 3rd order calculation needed to compute the next order symmetric matter fluctuation effect.

We express the S-matrix as

$$S = S_{00} + \sum_{n,m=0,1,2,\cdots} S^{(n,m)}_{01,1} , \tag{A.1}$$

where S_{00} is the 0th order term from H_{00}, $S^{(n,m)}_{01,1}$ represents the (n+m)th order term from $H^n_{01} \times H^m_1$, i.e., the term of order $(\Delta m^2_{21}L/4E)^n(\delta aL/4E)^m$.

The contributions to T violation is summarized as follows: (1) The 1st order term is from $S_{00}S^{(1,0)*}_{01,1}$ and is proportional to $\sin\delta$ and contains the contribution from the constant (average) matter. (2) The 2nd order term is from $S_{00}S^{(1,1)*}_{01,1}$ and $S_{01}S^{(0,1)*}_{01,1}$. The symmetric and asymmetric matter fluctuations contribute to $\sin\delta$ and $\cos\delta$ terms, respectively. (3) The 2nd order term, $S_{00}S^{(0,2)*}_{01,1}$ and the 3rd order term, $S_{00}S^{(0,3)*}_{01,1}$ do not contribute to T violation.

Firstly, we present the result given in Ref.1:

$$\begin{aligned} & S_{00} = e^{-iH_{00}L} = \tilde{U}_0 P(L) \tilde{U}_0^\dagger \\ = \; & \frac{1}{2}\begin{pmatrix} \phi_+ - c_{2\tilde{z}}\phi_- & s_y s_{2\tilde{z}} e^{-i\delta}\phi_- & c_y s_{2\tilde{z}} e^{-i\delta}\phi_- \\ s_y s_{2\tilde{z}} e^{i\delta}\phi_- & \phi_+ + s_y^2 c_{2\tilde{z}}\phi_- - c_y^2(\phi_{2-} - \phi_{1-}) & s_{2y}\left(c^2_{\tilde{z}}\phi_{2-} - s^2_{\tilde{z}}\phi_{1-}\right) \\ c_y s_{2\tilde{z}} e^{i\delta}\phi_- & s_{2y}\left(c^2_{\tilde{z}}\phi_{2-} - s^2_{\tilde{z}}\phi_{1-}\right) & \phi_+ + c_y^2 c_{2\tilde{z}}\phi_- - s_y^2(\phi_{2-} - \phi_{1-}) \end{pmatrix} , \end{aligned} \tag{A.2}$$

where $\phi_\pm$ and $\phi_{i\pm}$ $(i = 1, 2)$ are defined as follows.

$$\begin{aligned} \phi_\pm &= e^{-ia_+L} \pm e^{-ia_-L} , \\ \phi_{1\pm} &= e^{-ia_0L} \pm e^{-ia_-L} , \quad \phi_{2\pm} = e^{-ia_+L} \pm e^{-ia_0L} . \end{aligned} \tag{A.3}$$

$S_{01,1}^{(n,m)}(n \geq 1)$ is parametrized by

$$
\begin{aligned}
S_{01,1}^{(n,m)} &= \tilde{U}_0 \begin{pmatrix} 0 & \mathcal{A}^{(n,m)} & 0 \\ \mathcal{A}'^{(n,m)} & 0 & \mathcal{B}^{(n,m)} \\ 0 & \mathcal{B}'^{(n,m)} & 0 \end{pmatrix} \tilde{U}_0^\dagger \\
&= \begin{pmatrix} 0 & c_y \mathcal{P}^{(n,m)} & -s_y \mathcal{P}^{(n,m)} \\ c_y \mathcal{P}'^{(n,m)} & -s_y c_y (e^{i\delta} \mathcal{Q}^{(n,m)} + e^{-i\delta} \mathcal{Q}'^{(n,m)}) & e^{i\delta} s_y^2 \mathcal{Q}^{(n,m)} - e^{-i\delta} c_y^2 \mathcal{Q}'^{(n,m)} \\ -s_y \mathcal{P}'^{(n,m)} & -e^{i\delta} c_y^2 \mathcal{Q}^{(n,m)} + e^{-i\delta} s_y^2 \mathcal{Q}'^{(n,m)} & s_y c_y (e^{i\delta} \mathcal{Q}^{(n,m)} + e^{-i\delta} \mathcal{Q}'^{(n,m)}) \end{pmatrix} ,
\end{aligned}
\tag{A.4}
$$

where

$$
\begin{aligned}
\mathcal{P}^{(n,m)} &= (c_{\tilde{z}} \mathcal{A}^{(n,m)} + s_{\tilde{z}} \mathcal{B}^{(n,m)}) , \ \mathcal{Q}^{(n,m)} = (s_{\tilde{z}} \mathcal{A}^{(n,m)} - c_{\tilde{z}} \mathcal{B}^{(n,m)}) , \\
\mathcal{P}'^{(n,m)} &= (c_{\tilde{z}} \mathcal{A}'^{(n,m)} + s_{\tilde{z}} \mathcal{B}'^{(n,m)}) , \ \mathcal{Q}'^{(n,m)} = (s_{\tilde{z}} \mathcal{A}'^{(n,m)} - c_{\tilde{z}} \mathcal{B}'^{(n,m)}) .
\end{aligned}
\tag{A.5}
$$

The $S_{01,1}^{(1,0)}$ is obatained from

$$
\begin{aligned}
\mathcal{A}^{(1,0)} &= \mathcal{A}'^{(1,0)} = \frac{\Delta m_{21}^2}{4E} \frac{s_{2x} c_{z-\tilde{z}}}{k_1} \phi_{1-} , \\
\mathcal{B}^{(1,0)} &= \mathcal{B}'^{(1,0)} = -\frac{\Delta m_{21}^2}{4E} \frac{s_{2x} s_{z-\tilde{z}}}{k_2} \phi_{2-} .
\end{aligned}
\tag{A.6}
$$

The $S_{01,1}^{(1,1)}$ is obatained from

$$
\begin{aligned}
\mathcal{A}^{(1,1s)} &= \mathcal{A}'^{(1,1s)} \\
&= \frac{\Delta m_{21}^2 s_{2x}}{4E} c_{\tilde{z}} \left\{ \left(\frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} + \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} \right) \right. \\
&\quad \left. \times \left(\sum_{n=1,2,\cdots} \frac{a_{2n} k_1}{E(k_1^2 - q_{2n}^2)} \right) \phi_{1-} - \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} \sum_{n=1,2,\cdots} \left(\frac{a_{2n} k}{E(k^2 - q_{2n}^2)} \right) \phi_{-} \right\} , \\
\mathcal{B}^{(1,1s)} &= \mathcal{B}'^{(1,1s)} \\
&= \frac{\Delta m_{21}^2 s_{2x}}{4E} s_{\tilde{z}} \left\{ \left(\frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} + \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} \right) \right. \\
&\quad \left. \times \left(\sum_{n=1,2,\cdots} \frac{a_{2n} k_2}{E(k_2^2 - q_{2n}^2)} \right) \phi_{2-} - \frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} \left(\sum_{n=1,2,\cdots} \frac{a_{2n} k}{E(k^2 - q_{2n}^2)} \right) \phi_{-} \right\} ,
\end{aligned}
\tag{A.7}
$$

for symmetric matter fluctuations and

$$
\mathcal{A}^{(1,1a)} = -\mathcal{A}'^{(1,1a)}
$$

$$
\begin{aligned}
&= -\frac{\Delta m_{21}^2 s_{2x}}{4E} c_{\tilde{z}} \left\{ \left(\frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} + \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} \right) \right. \\
&\quad \left. \times \left(\sum_{n=0,1,\cdots} \frac{a_{2n+1} k_1}{E(k_1^2 - q_{2n+1}^2)} \right) \phi_{1-} - \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} \sum_{n=0,1,\cdots} \left(\frac{a_{2n+1} k}{E(k^2 - q_{2n+1}^2)} \right) \phi_{-} \right\} , \\
\mathcal{B}^{(1,1a)} &= -\mathcal{B}'^{(1,1a)} \\
&= \frac{\Delta m_{21}^2 s_{2x}}{4E} s_{\tilde{z}} \left\{ \left(\frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} + \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} \right) \right. \\
&\quad \left. \times \left(\sum_{n=0,1,\cdots} \frac{a_{2n+1} k_2}{E(k_2^2 - q_{2n+1}^2)} \right) \phi_{2-} - \frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} \left(\sum_{n=0,1,\cdots} \frac{a_{2n+1} k}{E(k^2 - q_{2n+1}^2)} \right) \phi_{-} \right\} , \qquad \text{(A.8)}
\end{aligned}
$$

for asymmetric fluctuations.

$S_{01,1}^{(0,n)}$ is given by

$$
S_{01,1}^{(0,n)} = \tilde{U}_0 \begin{pmatrix} \mathcal{E}^{(0,n)} & 0 & \mathcal{C}^{(0,n)} \\ 0 & 0 & 0 \\ \mathcal{D}^{(0,n)} & 0 & \mathcal{F}^{(0,n)} \end{pmatrix} \tilde{U}_0^\dagger = \frac{1}{2} \begin{pmatrix} \alpha_n^{(+)} & e^{-i\delta} s_y \beta_n^{(+)} & e^{-i\delta} c_y \beta_n^{(+)} \\ e^{i\delta} s_y \beta_n^{(-)} & s_y^2 \alpha_n^{(-)} & s_y c_y \alpha_n^{(-)} \\ e^{i\delta} c_y \beta_n^{(-)} & s_y c_y \alpha_n^{(-)} & c_y^2 \alpha_n^{(-)} \end{pmatrix} , \qquad \text{(A.9)}
$$

where

$$
\begin{aligned}
\alpha_n^{(\pm)} &= \mathcal{E}^{(0,n)} + \mathcal{F}^{(0,n)} \pm \left(c_{2\tilde{z}} (\mathcal{E}^{(0,n)} - \mathcal{F}^{(0,n)}) + s_{2\tilde{z}} (\mathcal{C}^{(0,n)} + \mathcal{D}^{(0,n)}) \right) , \\
\beta_n^{(\pm)} &= -s_{2\tilde{z}} (\mathcal{E}^{(0,n)} - \mathcal{F}^{(0,n)}) + c_{2\tilde{z}} (\mathcal{C}^{(0,n)} + \mathcal{D}^{(0,n)}) \pm (\mathcal{C}^{(0,n)} - \mathcal{D}^{(0,n)}) . \qquad \text{(A.10)}
\end{aligned}
$$

$S_{01,1}^{(0,1)}$ is derived from

$$
\begin{aligned}
\mathcal{C}^{(0,1)} + \mathcal{D}^{(0,1)} &= \frac{s_{2\tilde{z}}}{2E} \left(\sum_{n \neq 0} \frac{a_{2n}}{k + q_{2n}} \right) \phi_{-} , \\
\mathcal{C}^{(0,1)} - \mathcal{D}^{(0,1)} &= -\frac{s_{2\tilde{z}}}{2E} \left(\sum_{n} \frac{a_{2n+1}}{k + q_{2n+1}} \right) \phi_{+} , \\
\mathcal{E}^{(0,1)} &= \mathcal{F}^{(0,1)} = 0 . \qquad \text{(A.11)}
\end{aligned}
$$

By using the above fromula, we obtained the 2nd order formula for T violation.

Finally, we give the result for $S_{01,1}^{(0,2)}$ which is needed to estimate the 3rd order correction. $S_{01,1}^{(0,2)}$ is calculated from

$$
\begin{aligned}
\mathcal{C}^{(0,2)} + \mathcal{D}^{(0,2)} &= \frac{s_{2\tilde{z}} c_{2\tilde{z}}}{(2E)^2} \left(\sum_{m+n=\text{even}} \frac{a_n a_m}{(k + q_n)(k + q_n + q_m)} \right) \phi_{-} , \\
\mathcal{C}^{(0,2)} - \mathcal{D}^{(0,2)} &= -\frac{s_{2\tilde{z}} c_{2\tilde{z}}}{(2E)^2} \left(\sum_{m+n=\text{odd}} \frac{a_n a_m}{(k + q_n)(k + q_n + q_m)} \right) \phi_{+} ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{E}^{(0,2)} - \mathcal{F}^{(0,2)} &= \frac{s_{2\tilde{z}}^2}{8(2E)^2} \left\{ \left(\sum n, m \frac{((-1)^n + 1)((-1)^m + 1) a_n a_m}{(k + q_n)(k + q_m)} \right) \phi_- \right. \\
&\qquad \left. + 2L \left(\sum_n \frac{a_n a_{-n}}{k + q_n} \right) i\phi_+ \right\} . \qquad \text{(A.12)}
\end{aligned}
$$

Appendix B : The 3rd order calculation

We consider the 3rd order correction form $H_{01} H_1^2$ where we take symmetric matter fluctuations for H_1. The contribution arises from $S_{00} S_{01,1}^{(1,2)*}$, $S_{01,1}^{(1,0)} S_{01,1}^{(0,2)*}$ and $S_{01,1}^{(1,1)} S_{01,1}^{(0,1)*}$. The latter two can be calculated the result given in Appendix A. Here, we give the result for $S_{01,1}^{(1,2)}$ which is given from

$$
\begin{aligned}
\mathcal{A}^{(1,2s)} &= \mathcal{A}'^{(1,2s)} = \frac{\Delta m_{21}^2 s_{2x}}{4E} c_{\tilde{z}} \\
&\times \left\{ \left(\frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} + \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} \right) \left(\sum_{n \neq 0} \frac{a_{2n} a_{2m}}{(2E)^2} \left(\frac{c_{\tilde{z}}^2}{k_1 + q_{2m}} - \frac{s_{\tilde{z}}^2}{k_2 - q_{2m}} \right) \frac{1}{k_1 + q_{2n+2m}} \right) \phi_{1-} \right. \\
&\qquad \left. - \frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} s_{\tilde{z}}^2 L \left(\sum_{n \neq 0} \frac{a_{2n} a_{-2n}}{(2E)^2} \frac{1}{k + q_{2n}} \right) e^{-i a_- L} \right\} , \\
\mathcal{B}^{(1,2s)} &= \mathcal{B}'^{(1,2s)} = -\frac{\Delta m_{21}^2 s_{2x}}{4E} s_{\tilde{z}} \\
&\times \left\{ \left(\frac{c_{\tilde{z}} c_{z-\tilde{z}}}{k_1} + \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} \right) \left(\sum_{n \neq 0} \frac{a_{2n} a_{2m}}{(2E)^2} \left(\frac{c_{\tilde{z}}^2}{k_1 + q_{2m}} - \frac{s_{\tilde{z}}^2}{k_2 - q_{2m}} \right) \frac{1}{k_2 - q_{2n+2m}} \right) \phi_{1-} \right. \\
&\qquad \left. - \frac{s_{\tilde{z}} s_{z-\tilde{z}}}{k_2} c_{\tilde{z}}^2 L \left(\sum_{n \neq 0} \frac{a_{2n} a_{-2n}}{(2E)^2} \frac{1}{k + q_{2n}} \right) e^{-i a_+ L} \right\} .
\end{aligned}
\qquad \text{(B.1)}
$$

By using the above formula, we can derive the 3rd order contribution to T violation.

References

[1] T. Miura, E. Takasugi, Y. Kuno and M. Yoshimura, hep-ph/0102111, Phys. Rev. D **64**, 013002 (2001)

[2] D. Ayres *et al.* [Neutrino Factory and Muon Collider Collaboration], on a storage ring and a muon collider," physics/9911009 ;
C. Albright *et al.*, hep-ex/0008064.

[3] M. Koike and J. Sato, Mod. Phys. Lett. **A14** (1999) 1297.

[4] T. Ota and J. Sato, Phys. Rev. D **63**, 093004 (2001)

[5] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 247.

[6] A. M. Dziewonsky and D.L. Anderson, Phys. Earth. Planet. Inter. **25** (1981) 297;
A. Dziewonsky, "Earth Structure, Global," in *The Encyclopedia of Solid Earth Geophysics*, edited by David E. James (Van Nostrand Reinhold, New York, 1989), p. 331;
R. Gandhi, C. Quigg, M. Hall Reno and I. Sarcevic, Astropart. Phys. **5** (1996) 81.

[7] S. M. Bilenkii, J. Hosek and S. T. Petcov, Phys. Lett. B **94**, 495 (1980).
M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. B **102**, 323 (1981).

[8] P.I. Krastev and S.T. Petcov, Phys. Lett. B**205**, 84 (1988).

[9] E. K. Akhmedov, P. Huber, M. Lindner and T. Ohlsson, hep-ph/0105029.

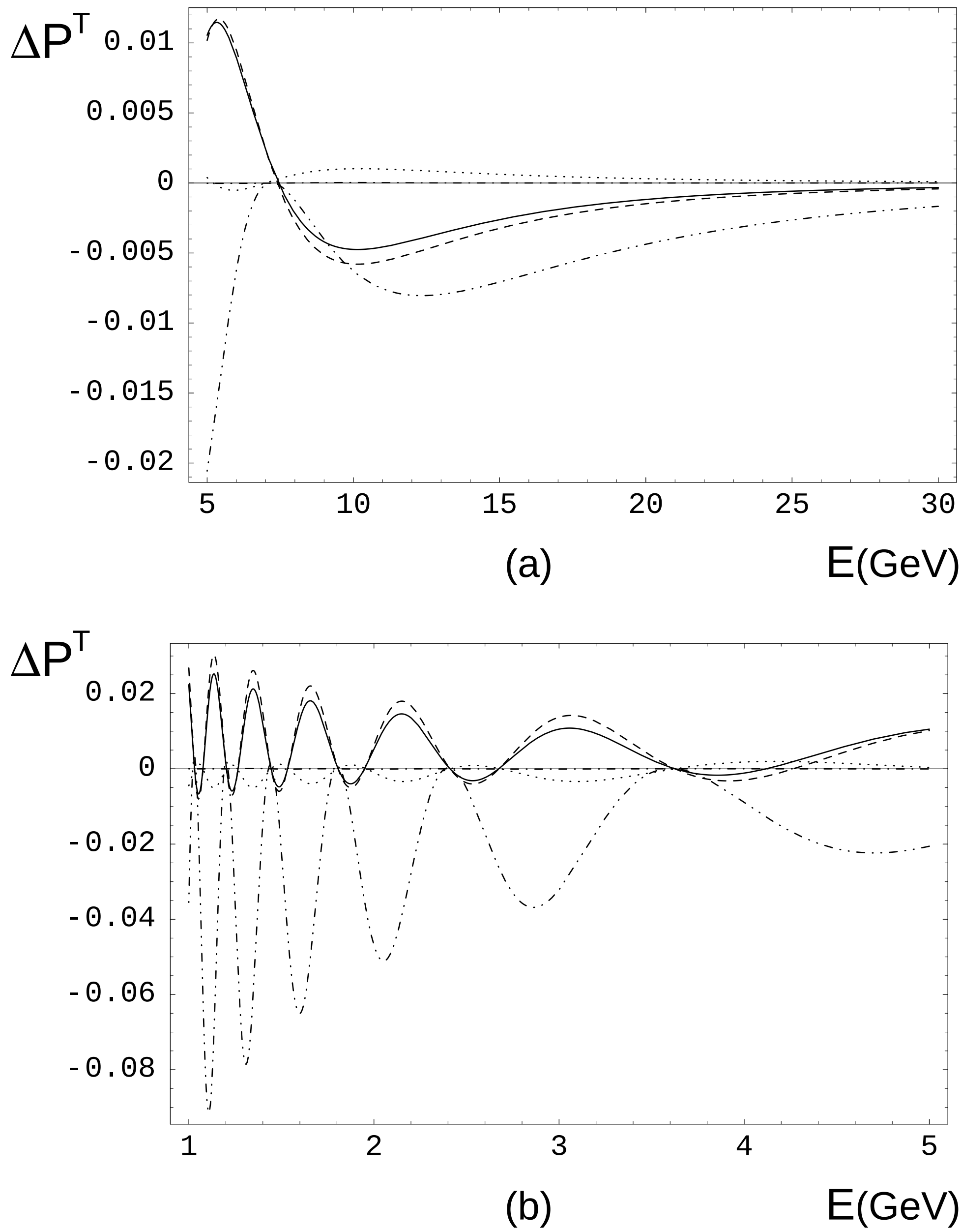


Figure 1: The energy dependence of the T violation, $\Delta P^T_{\nu_e\nu_\mu}$ with $L = 6000$km for (a) 5GeV< E <30GeV and (b) 1GeV< E <5GeV. The dashed line, the dotted line and the dash-dotted line show the 1st, the 2nd and the 3rd order terms. The solid line represents the sum of them. The vacuum (no matter) case is shown by the dash-twodotted line for the comparison. In this plot, we use $\sin 2\theta_x = \sin 2\theta_y = 1$, $\sin\theta_z = 0.1$, $\Delta m^2_{21} = 5\cdot 10^{-5}\mathrm{eV}^2$, $\Delta m^2_{31} = 3\cdot 10^{-3}\mathrm{eV}^2$ and $\delta = \pi/4$.

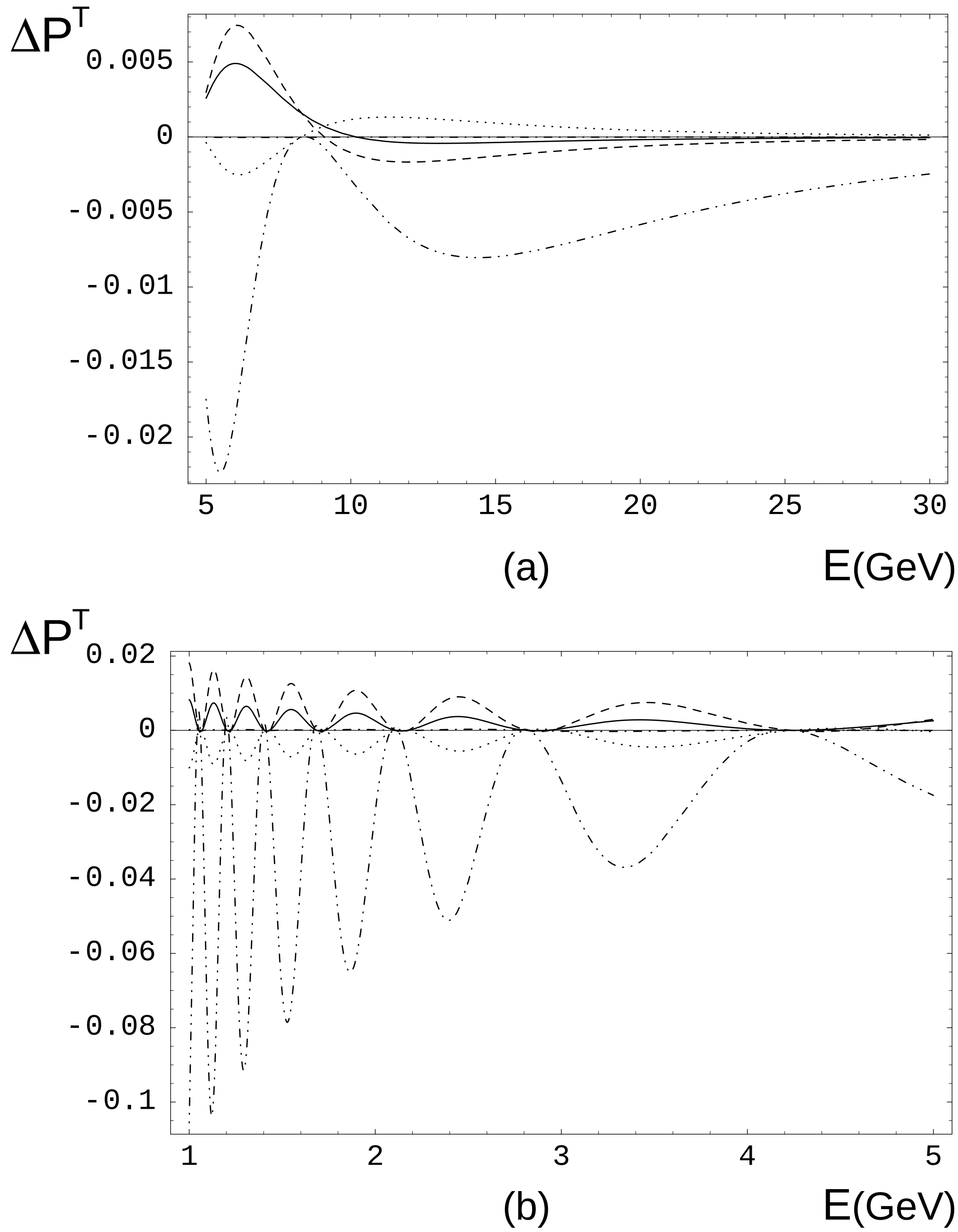


Figure 2: The energy dependence of the T violation, $\Delta P^T_{\nu_e\nu_\mu}$ with L = 7000km for (a) 5GeV< E <30GeV and (b) 1GeV< E <5GeV. The species of the lines and the oscillation parameters are the same as those in Fig.1.

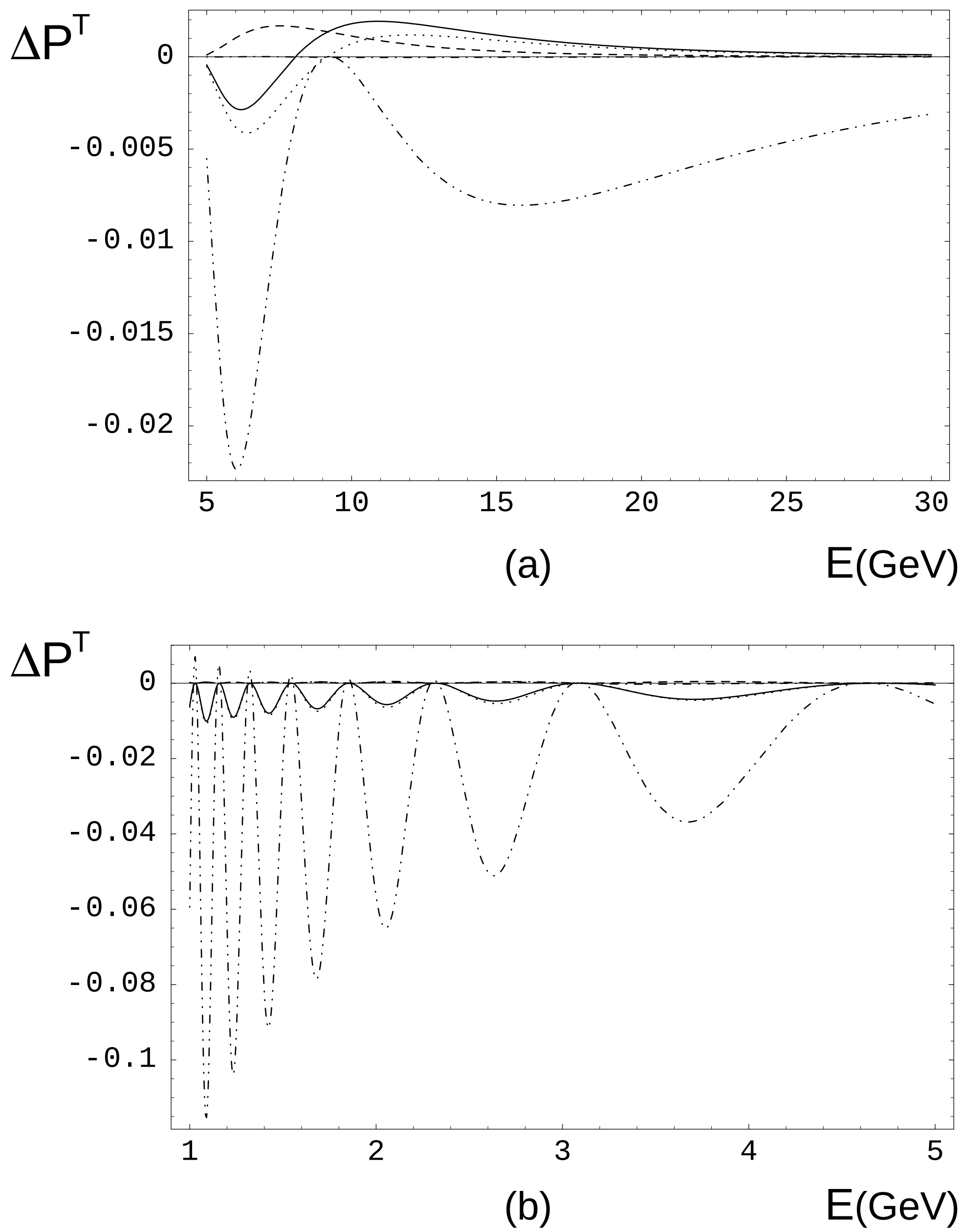


Figure 3: The energy dependence of the T violation, $\Delta P^{T}_{\nu_e\nu_\mu}$ with L = 7700km for (a) 5GeV< E <30GeV and (b) 1GeV< E <5GeV. The species of the lines and the oscillation parameters are the same as those in Fig.1.

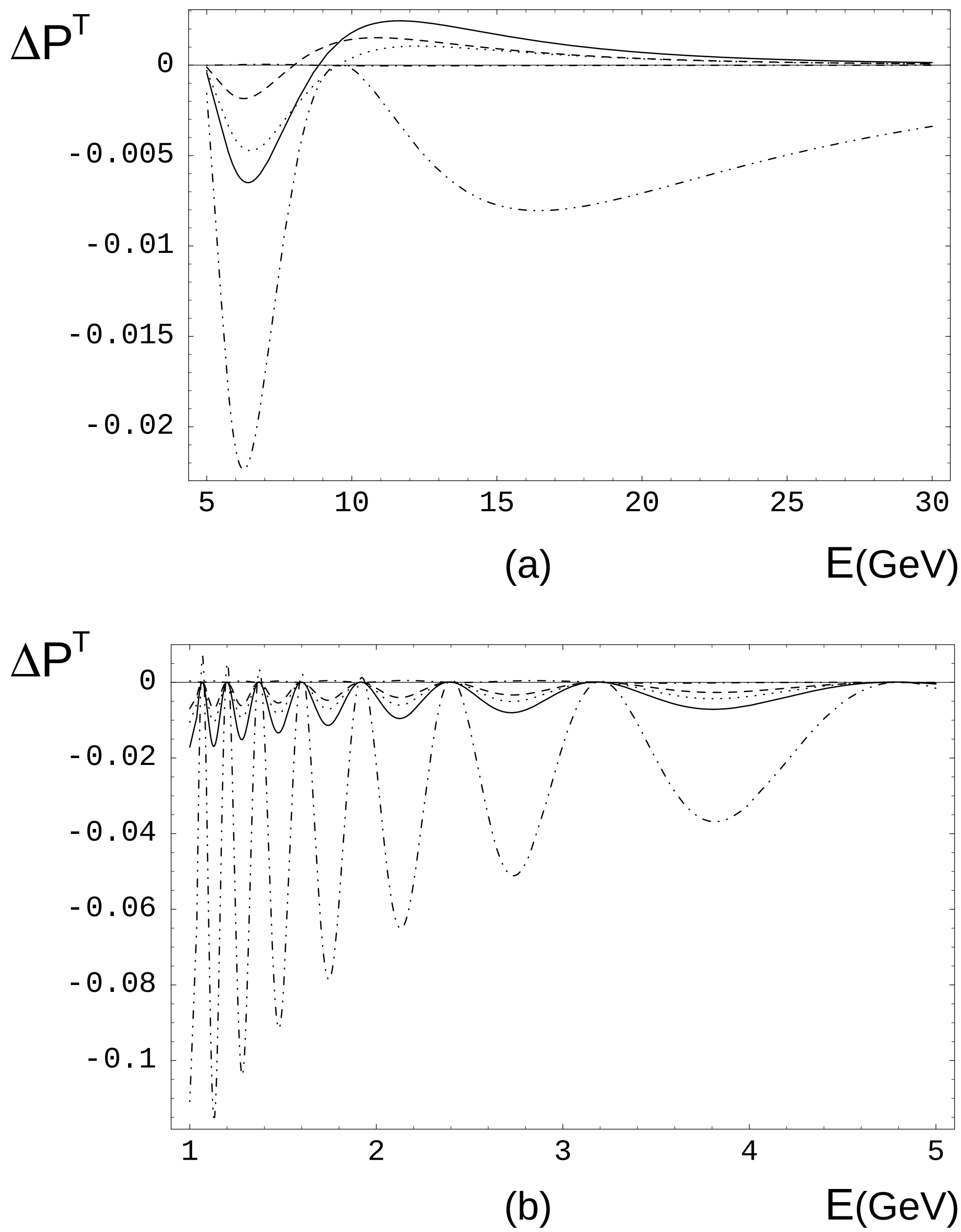


Figure 4: The energy dependence of the T violation, $\Delta P^T_{\nu_e\nu_\mu}$ with $L = 8000$km for (a) 5GeV< E <30GeV and (b) 1GeV< E <5GeV. The species of the lines and the oscillation parameters are the same as those in Fig.1.

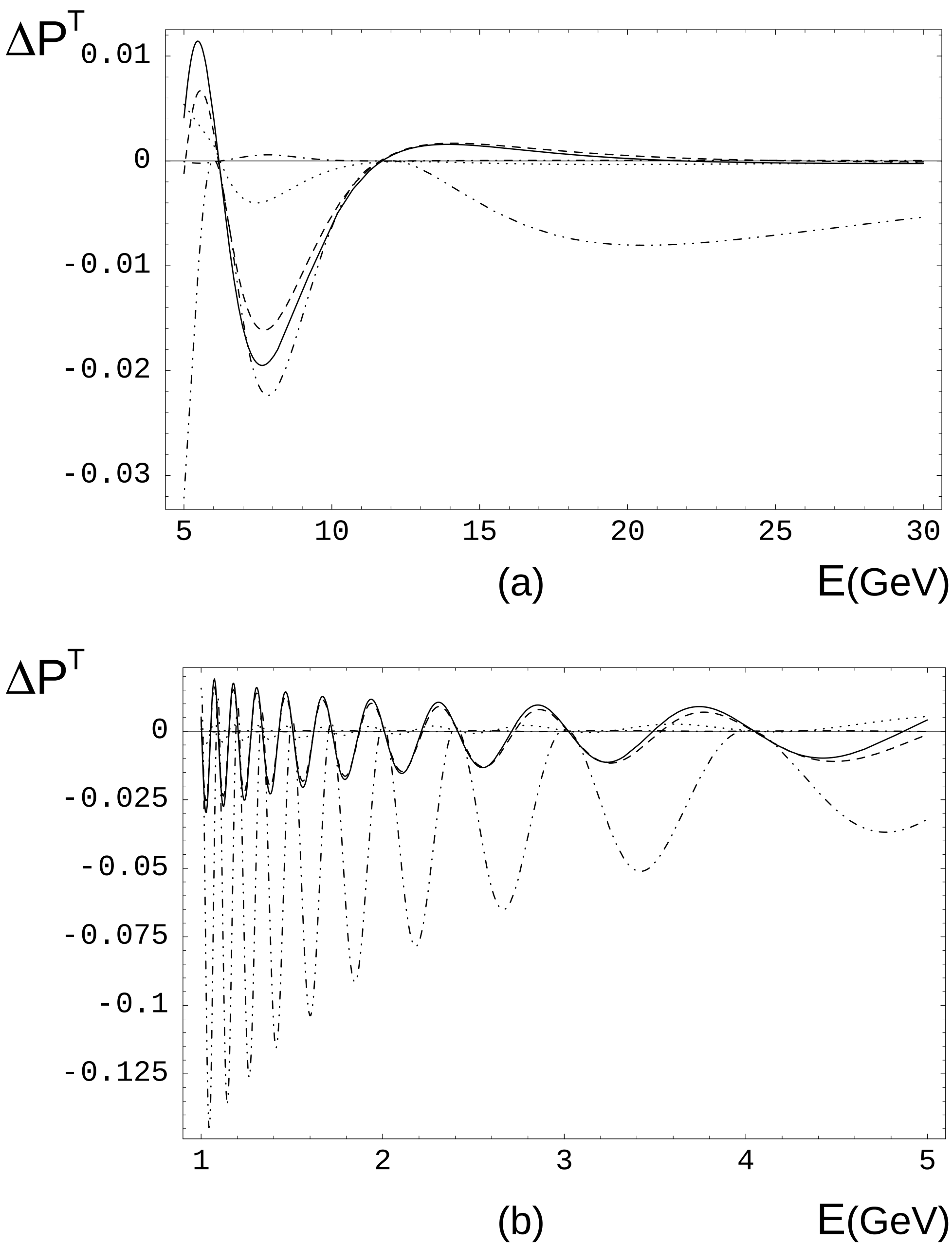


Figure 5: The energy dependence of the T violation, $\Delta P^{T}_{\nu_e\nu_\mu}$ with $L = 10000$km for (a) 5GeV$<E<$30GeV and (b) 1GeV$<E<$5GeV. The species of the lines and the oscillation parameters for (a) 5GeV$<E<$30GeV and (b) 1GeV$<E<$5GeVare the same as those in Fig.1.

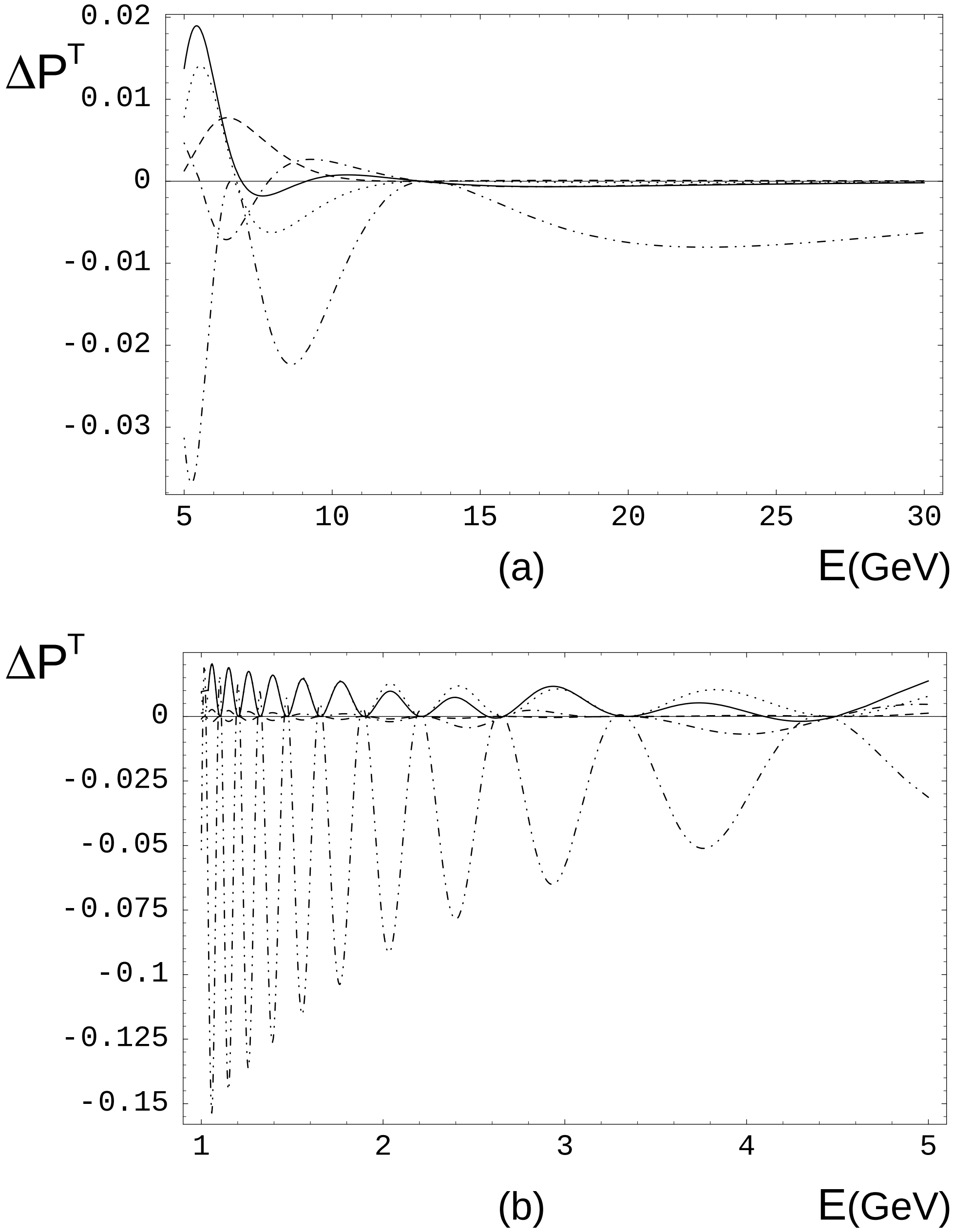
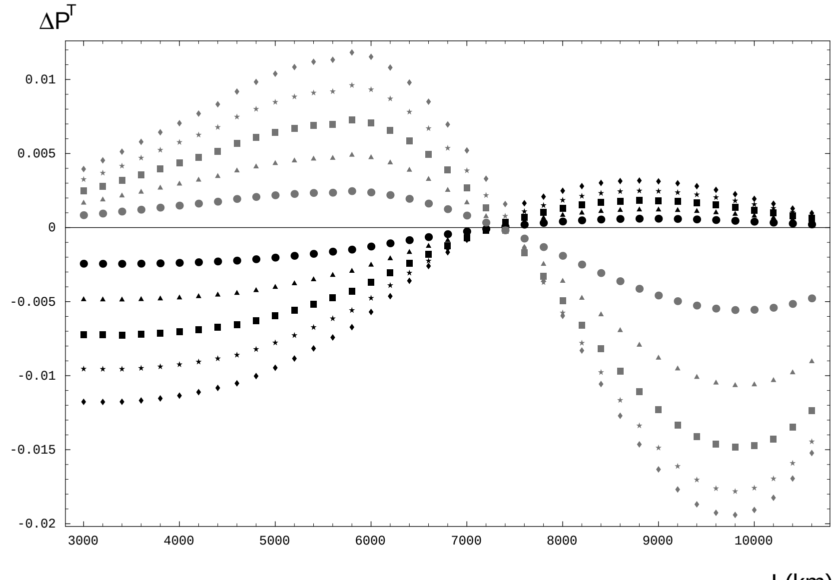
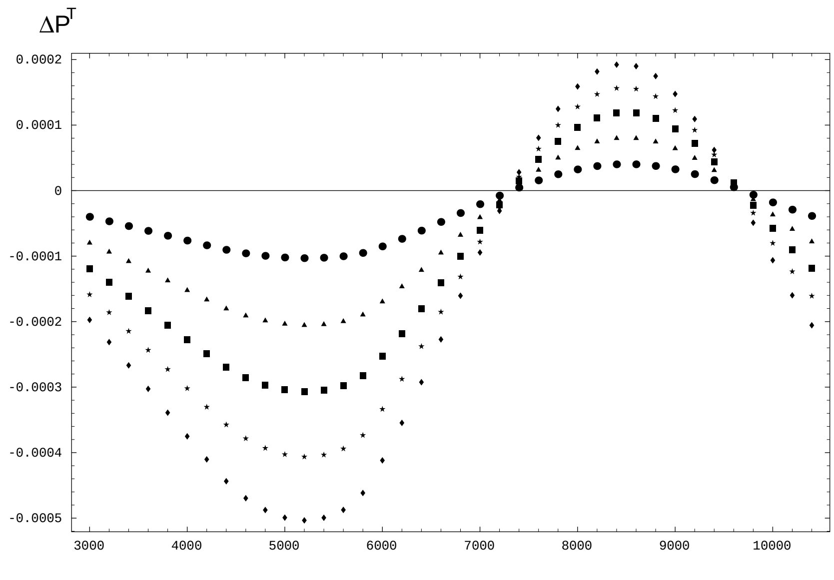


Figure 6: The energy dependence of the T violation, $\Delta P^T_{\nu_e\nu_\mu}$ with $L = 11000$km for (a) 5GeV< E <30GeV and (b) 1GeV< E <5GeV. The species of the lines and the oscillation parameters are the same as those in Fig.1.



(a)



(b)

Figure 7: The L dependence of T violation, $\Delta^T_{\nu_e\nu_\mu}$. In Fig.(a) T violation at the lower energy peak ($E = 5 \sim 8$ GeV) which is shown by faint points and the higher energy peak ($E = 10 \sim 13$ GeV) which is shown by dark points. Diamonds, stars, boxes, triangles and circles show T violation for $s_z = \sin\theta_{13} = 0.1$, 0.08, 0.06, 0.04 and 0.02 respectively. In Fig. (b) we show the values of T violation $\Delta P^T_{\nu_e\nu_\mu}$ at $E = 30$ GeV. The oscillation parameters except for s_z are the same as those in Fig.1.

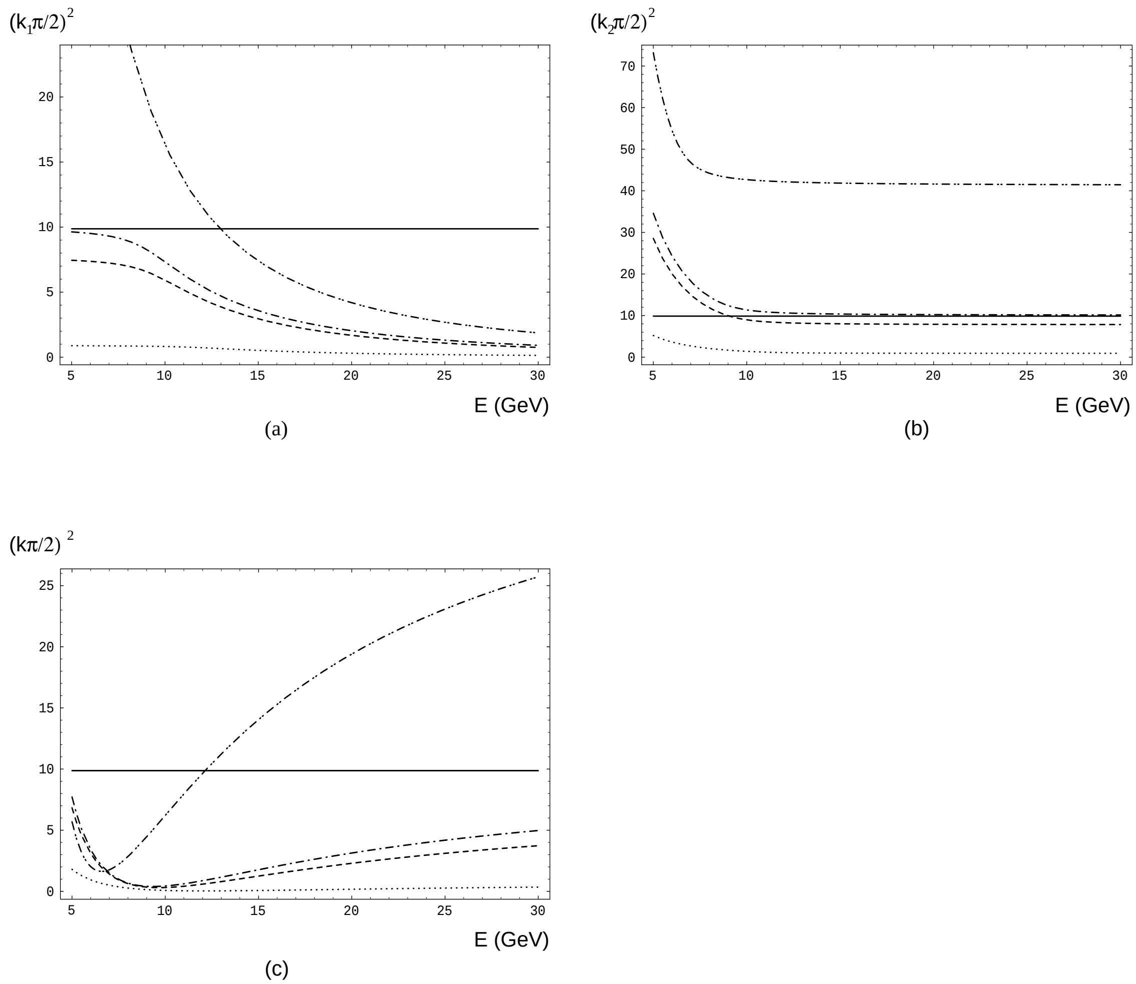


Figure 8: The energy dependence of $(k_1 L/2)^2$(Fig.(a)), $(k_2 L/2)^2$(Fig.(b)) and $(kL/2)^2$(Fig.(c)). Dotted, dashed, dash-dotted and dash-twodotted lines correspond to $L = 3000, 7000, 7700, 11000$ km, respectively. The solid line shows π^2. The oscillation parameters are the same as those in Fig.1.

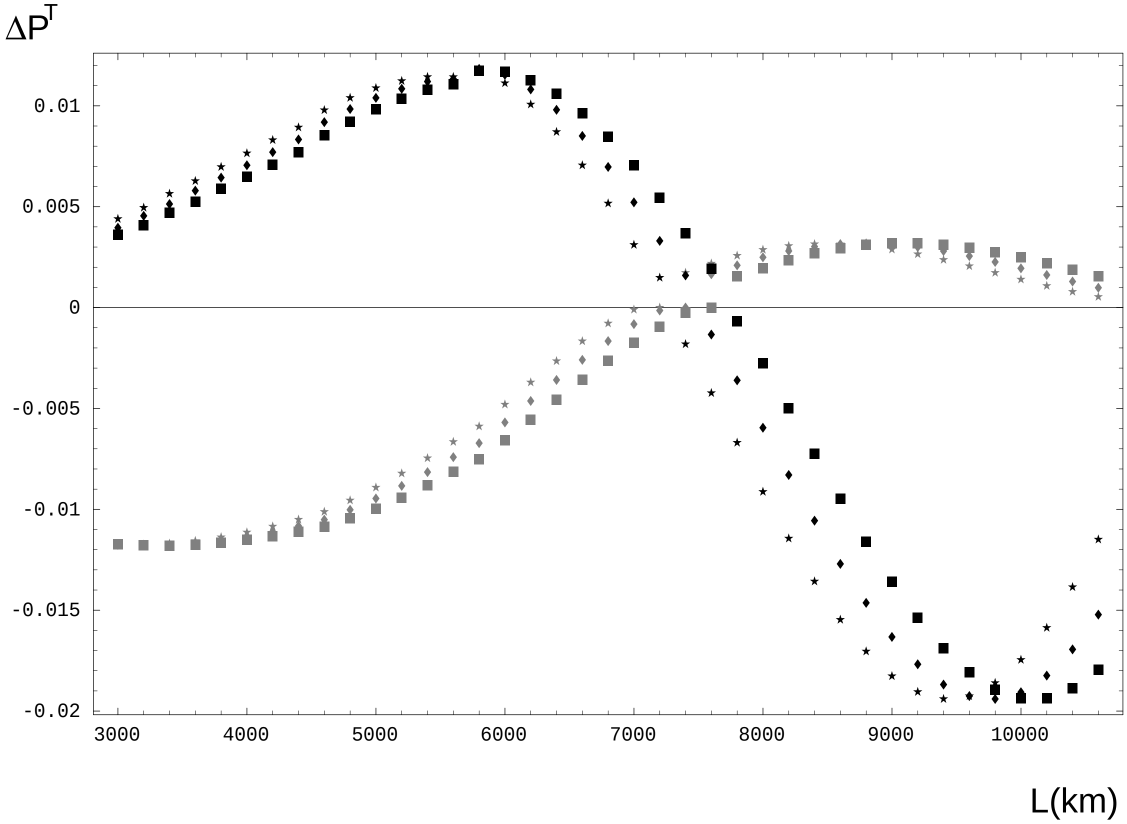


Figure 9: The average density ($\bar{a}$) dependence of T violation, $\Delta P^T_{\nu_e \nu_\mu}$. Diamonds, boxes and stars correspond to the average density value from PREM, 5% smaller value and 5% larger value. Faint and dark points correspond to the lower energy peak ($E = 5 \sim 8$ GeV) and the higher energy peak ($E = 10 \sim 13$ GeV). The oscillation parameters are the same as those in Fig.1.

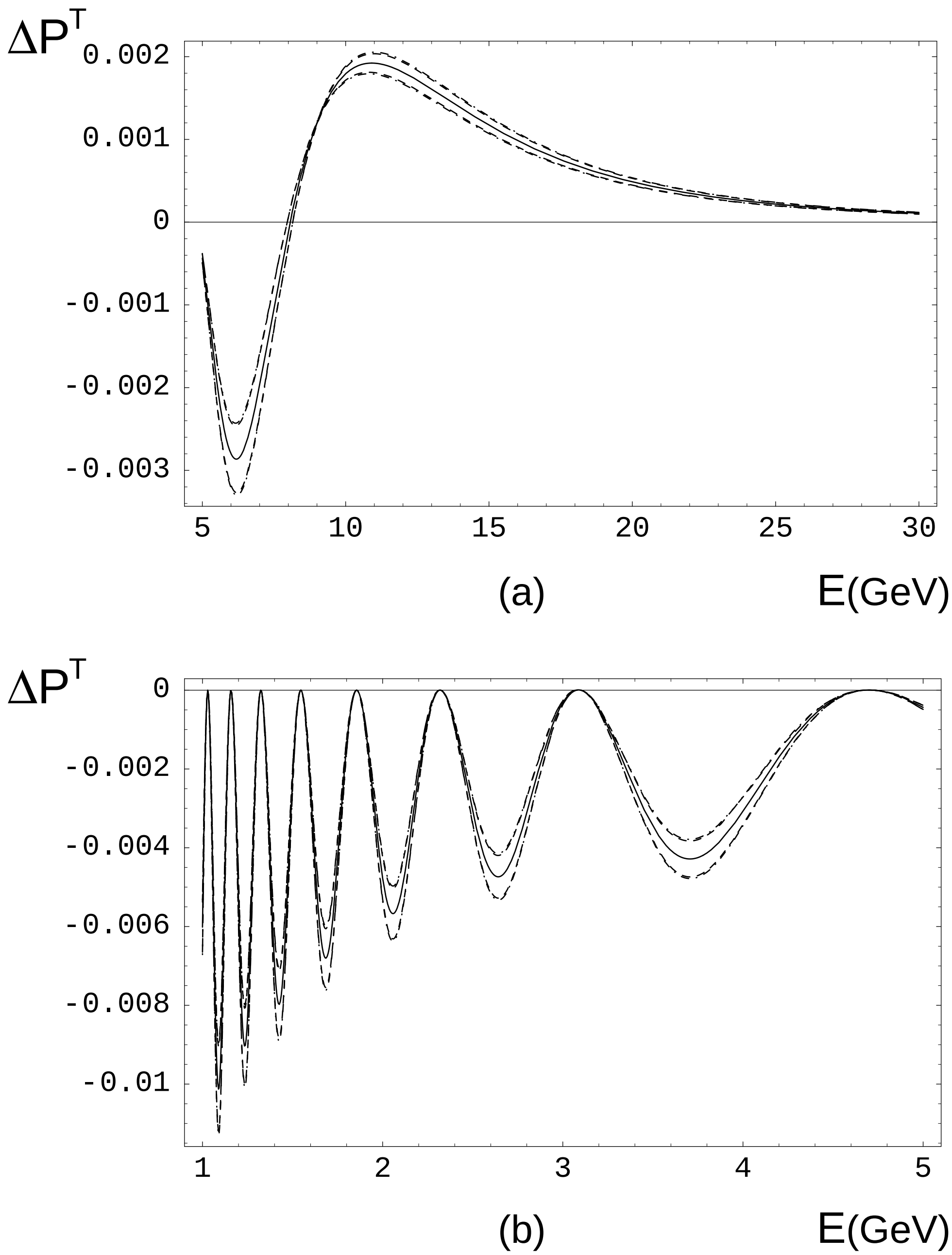


Figure 10: The variation of T violation, $\Delta P^T_{\nu_e \nu_\mu}$ by the deformation of the shape of matter profile for $L = 7700$km. Dotted lines show changes of T violation when Fourier coefficients for symmetric matter fluctuations a_{2n} are varied by 10% from PREM. Dash-dotted lines are those when asymmetric matter fluctuations are associated to be about 10% of PREM symmetric matter fluctuations. The oscillation parameters are the same as those in Fig.1.